LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering

Department of Electrical
and Computer Engineering

ELE 305: Introduction to Electrical Engineering
Exam 1 – Spring 2018

Duration: **1 hour 30 minutes**

Dr. Jihad Fahs

Start Time: 6:30 pm
Date: 13/3/2018

Name: ______________________________ **ID#:** ____________________

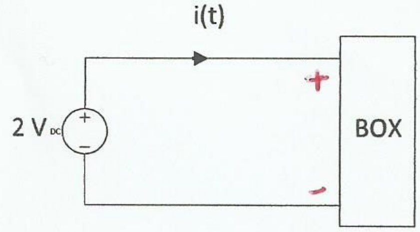
INSTRUCTIONS:

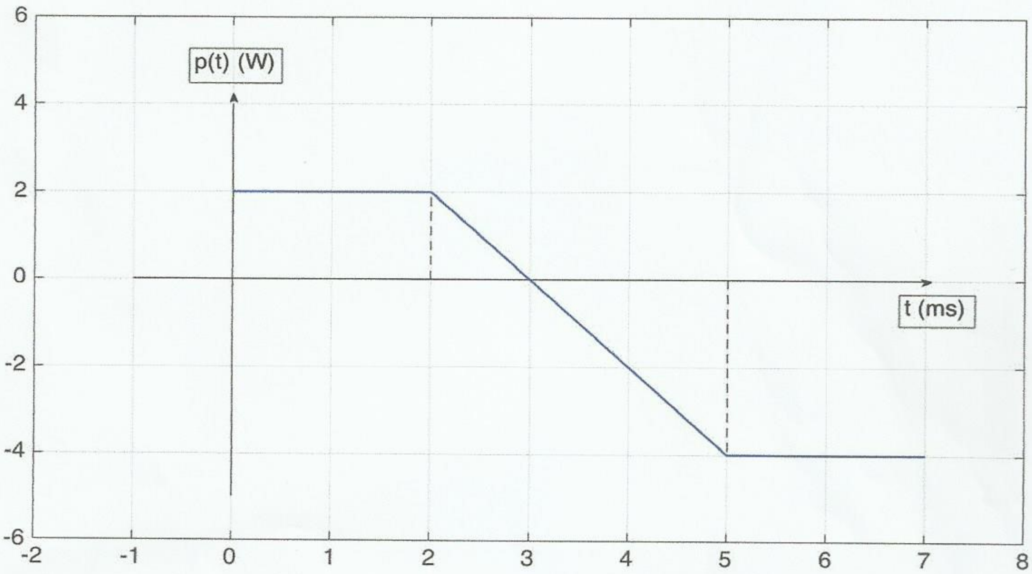
- Answer each of the following questions in the space provided. You can use both sides of each sheet for answers.
- This is a closed-book exam.
- If something is not clear, state your assumptions.
- Programmable calculators and smart devices are not allowed.
- The number of points for each question is specified next to it.
- The total number of points is 100.

1	2	3	4	5	6	Total
14	12	22	16	24	12	100

Question 1 [14 points]

The power absorbed by the box is shown in Figure 1. Find and sketch the charge q(t) that enters the box in the time interval 0<t<7 ms.





$$p(t) = v(t) \times i(t) \Rightarrow \underset{(A)}{i(t)} = \frac{1}{2}\,p(t) = \begin{cases} 1 & 0<t\leq 2 \\ -t+3 & 2<t\leq 5 \\ -2 & 5<t\leq 7 \end{cases}$$

$$q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0)$$

$\underline{0<t\leq 2:}$ $\quad q(t) = \int_0^t 1\,dt + q(0) = t$

at $t=2 \Rightarrow q(2) = 2$

$\underline{2<t\leq 5:}$ $\quad q(t) = \int_2^t (-t+3)\,dt + q(2)$

$$= -\frac{t^2}{2} + 3t\Big|_2^t + 2$$

$$= \frac{-t^2}{2} + 3t + \frac{4}{2} - 6 + 2$$

$$= -\frac{1}{2}t^2 + 3t - 2$$

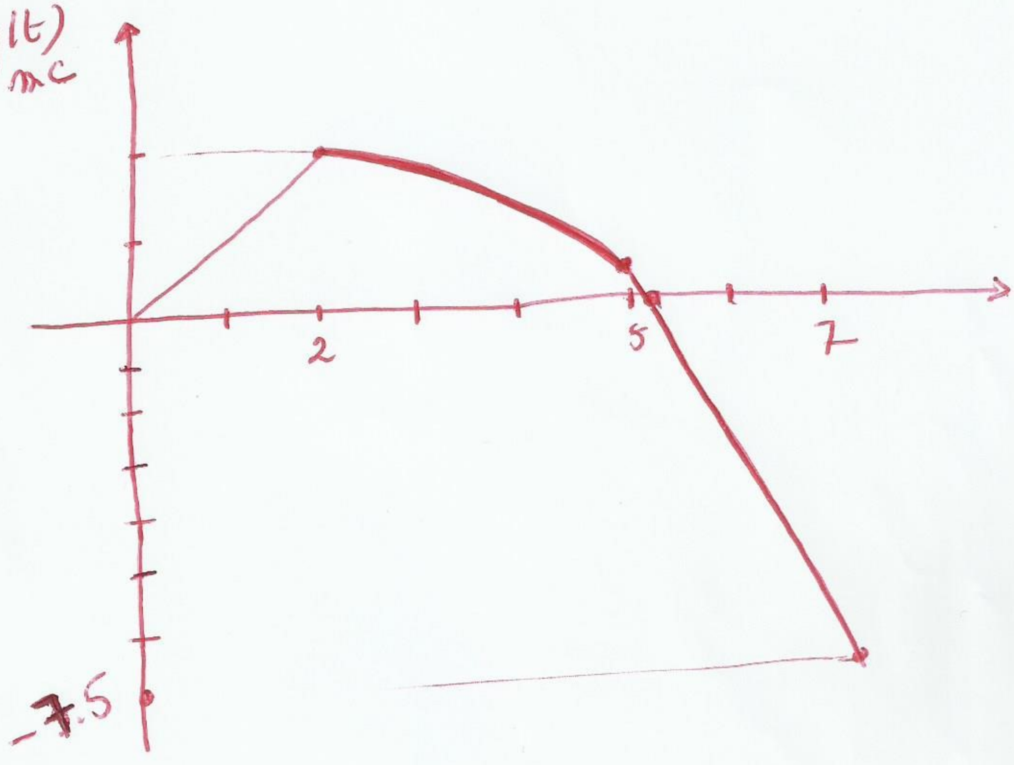
$\underline{\text{at } t=5:}$ $\quad q(5) = -\frac{1}{2}\times 25 + 15 - 2$

$$= \frac{1}{2}$$

$5 < t \leq 7$: $q(t) = \int_5^t i(t)\,dt + q(5) = \int_5^t -4\,dt + \frac{1}{2}$

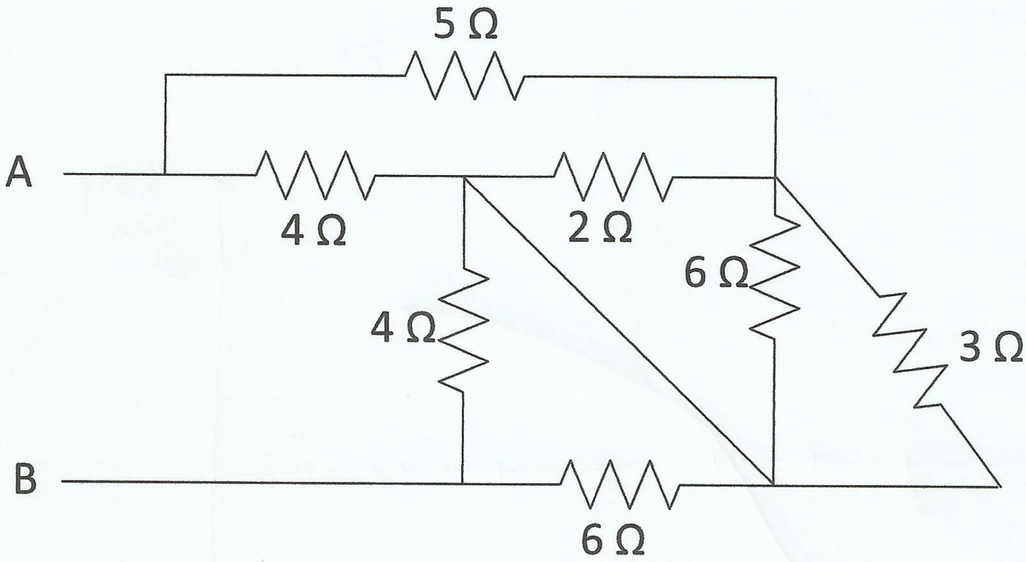
$= -4t \Big|_5^t + \frac{1}{2} = -4t + 20 + \frac{1}{2}$

$= -4t + 20.5$

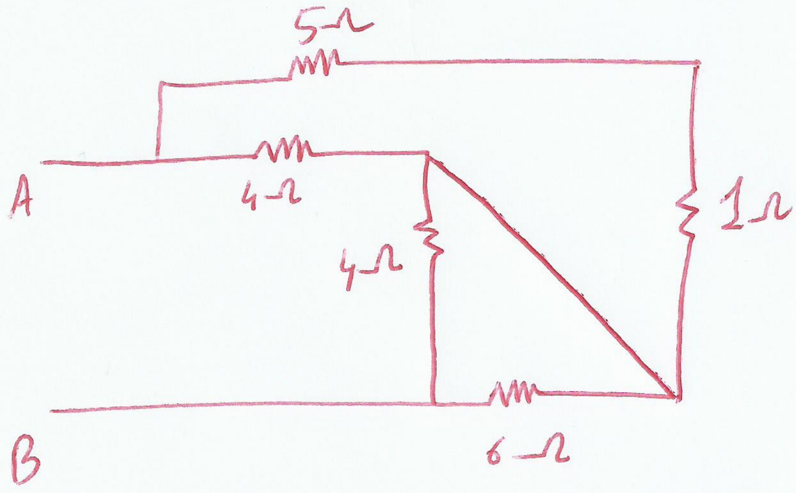


Question 2 [12 points]

Find R_{AB} in the network in the below figure.

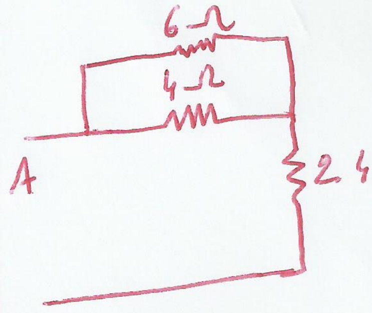


$3 // 6 // 2 : \rightarrow 1\Omega$

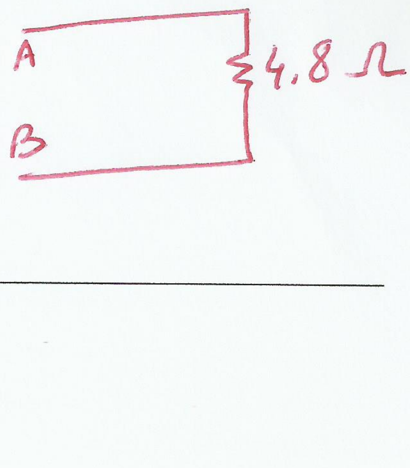


5 & 1 in Series → 6Ω

$4 // 6 \rightarrow 2.4\ \Omega$

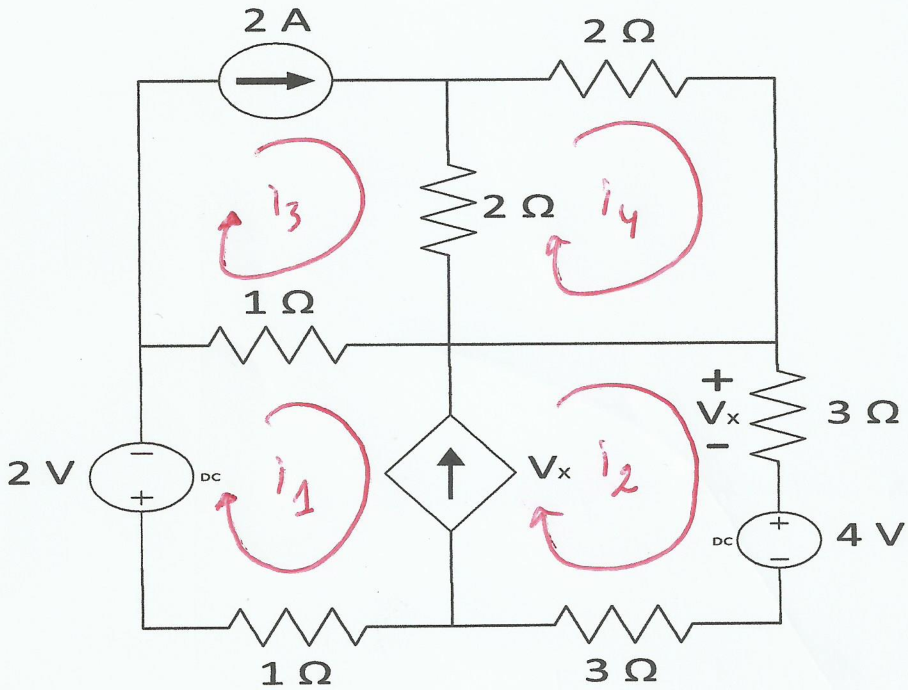


$6 // 4$ Series $2.4 \rightarrow 4.8$



Question 3 [22 points]

Use mesh analysis to calculate the power delivered by the 4V source in the network shown in the below figure.



Super-Mesh: $2 + 1(i_1 - i_3) + 3i_2 + 4 + 3i_2 + 1 \times i_1 = 0$ (1)

loop3: $\boxed{i_3 = 2A}$

loop4: $2(i_4 - i_3) + 2i_4 = 0 \Rightarrow 4i_4 = 2i_3 = 4 \Rightarrow \boxed{i_4 = 1A}$

Current Source: $i_2 - i_1 = V_x$

Controlling variable: $V_x = 3i_2$

$\Rightarrow \boxed{i_1 = -2i_2}$

(1) $\Rightarrow 2 + (i_1 - 2) + 6i_2 + 4 + i_1 = 0$

$\Rightarrow 2i_1 + 6i_2 + 4 = 0 \Rightarrow 2(-2i_2) + 6i_2 = -4 \Rightarrow$

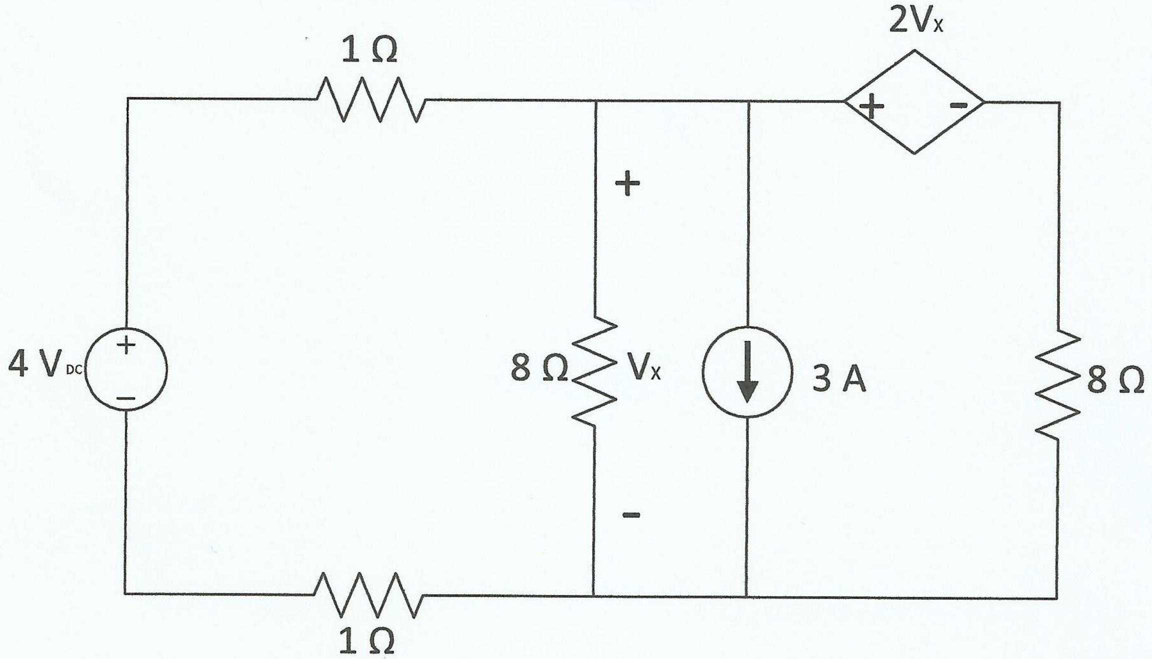
$2i_2 = -4 \Rightarrow \boxed{i_2 = -2A}$

and $i_1 = -2i_2 = 4A$

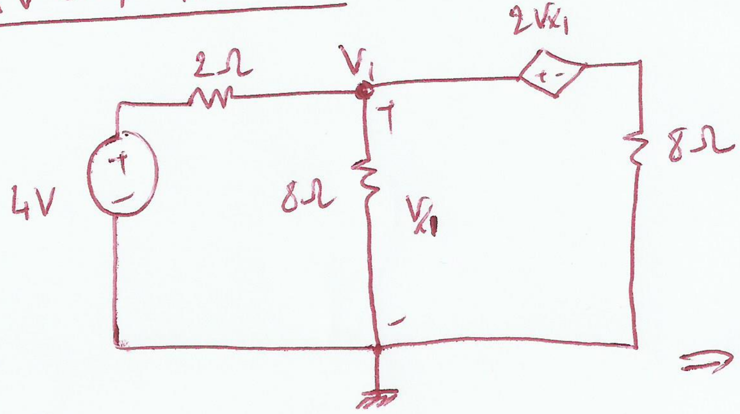
$P_{4V} = 4 \times i_2 = -8W$ (delivering 8 W)

Question 4 [16 points]

Consider the circuit shown below. Use superposition to find Vx.



4V ON, 3A OFF

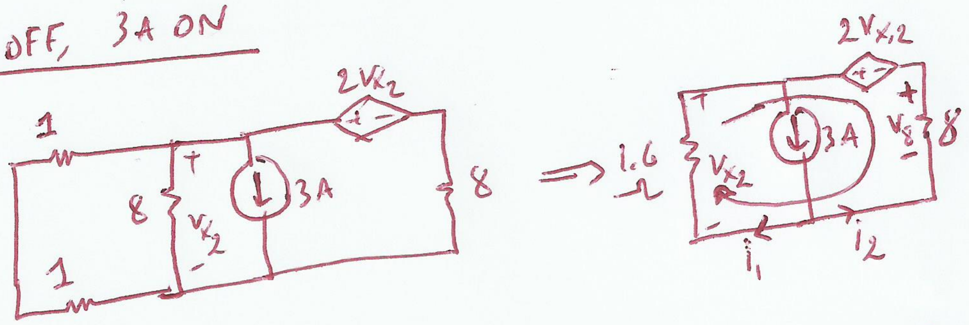


$$\begin{cases} \frac{V_1-4}{2} + \frac{V_1}{8} + \frac{V_1 - 2V_{x_1}}{8} = 0 \\ \text{Controlling variable: } V_1 = V_{x_1} \end{cases}$$

$$\Rightarrow \frac{V_1}{2} + \frac{V_1}{8} + \frac{V_1 - 2V_1}{8} = 2 \Rightarrow$$

$$\boxed{V_1 = 4V = V_{x_1}}$$

4V OFF, 3A ON



KVL: $-V_{x,2} + 2V_{x,2} + V_8 = 0$

$\Rightarrow V_8 = -V_{x,2}$ (1)

KCL: $i_1 + i_2 = 3$ (2)

ohm's law: $V_{x_2} = -1.6\, i_1$

$V_8 = -8\, i_2$

(1) $\Rightarrow -8i_2 - 1.6 i_1 = 0 \Rightarrow$

$i_2 = -\frac{1.6}{8} i_1 = -0.2 i_1$

(2) $\Rightarrow i_1 - 0.2 i_1 = 3 \Rightarrow i_1 = \frac{3}{0.8} = 3.75\,A$

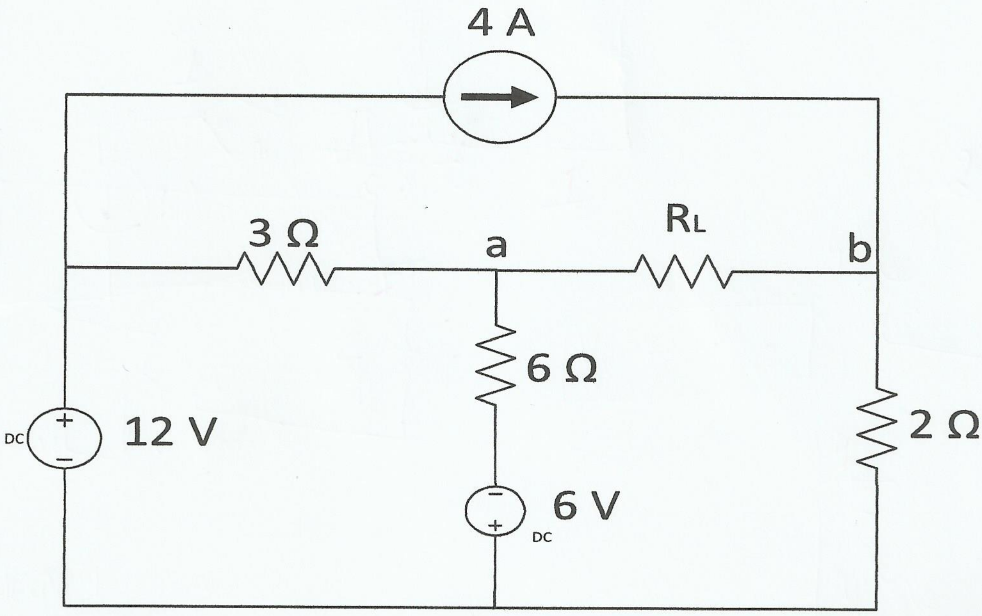
$\Rightarrow V_{x_2} = -1.6 \times i_1 = -6V$

Finally $V_x = V_{x_1} + V_{x_2} = 4 - 6 = -2V$

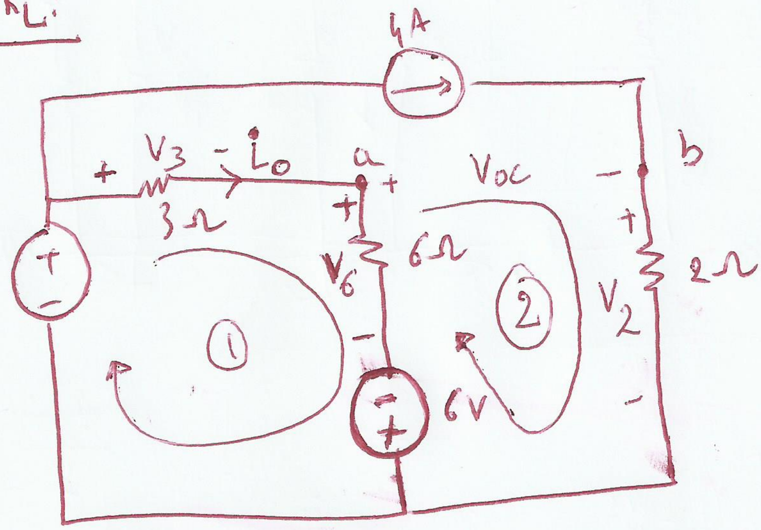
Question 5 [22 points]

Consider the network in the figure below.

a. Find the Thevenin equivalent of the circuit between nodes a and b as seen by the resistance R_L.
b. The resistance R_L absorbs 80% of the maximum power transfer. Find R_L.



Removing R_L:



Finding V_{Th}: $V_{Th} = V_{OC} = V_{ab}$

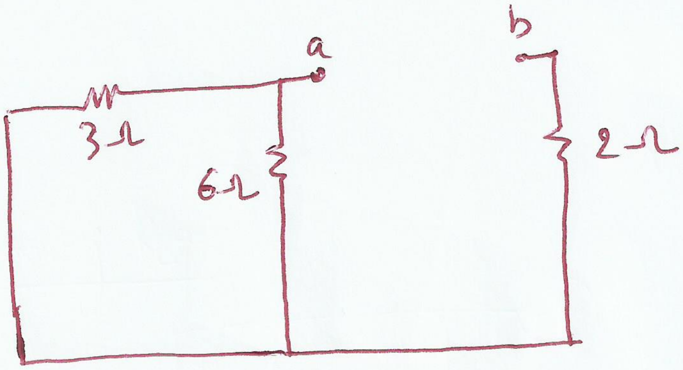
KVL in ①: $-12 + 3i_o + 6i_o - 6 = 0 \Rightarrow 9i_o = 18 \Rightarrow i_o = 2A$

KVL in ②: $+6 - V_6 + V_{OC} + V_2 = 0$

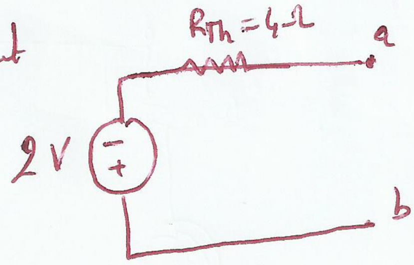
$$\Longrightarrow V_{OC} = V_6 - V_2 - 6$$
$$= 6 i_0 - 2 \times 4 - 6$$
$$= 6 \times 2 - 2 \times 4 - 6$$
$$= -2V$$

Finding R_{Th}: Turning off the Sources

$$R_{Th} = R_{ab} = R_{eq} = 3 // 6 + 2$$
$$= 4\Omega$$



$\Rightarrow$ the equivalent circuit

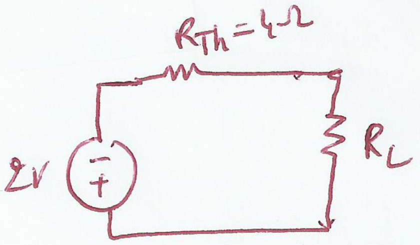


b. We know that

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{4}{4\times 4} = \frac{1}{4}\,W$$

$$P_{R_L} = \frac{V_{R_L}^2}{R_L} = \left(2 \times \frac{R_L}{R_L + R_{Th}}\right)^2 \times \frac{1}{R_L}$$

$$= 4 \frac{R_L^2}{(R_L+4)^2} \times \frac{1}{R_L} = \frac{4R_L}{(R_L+4)^2}$$



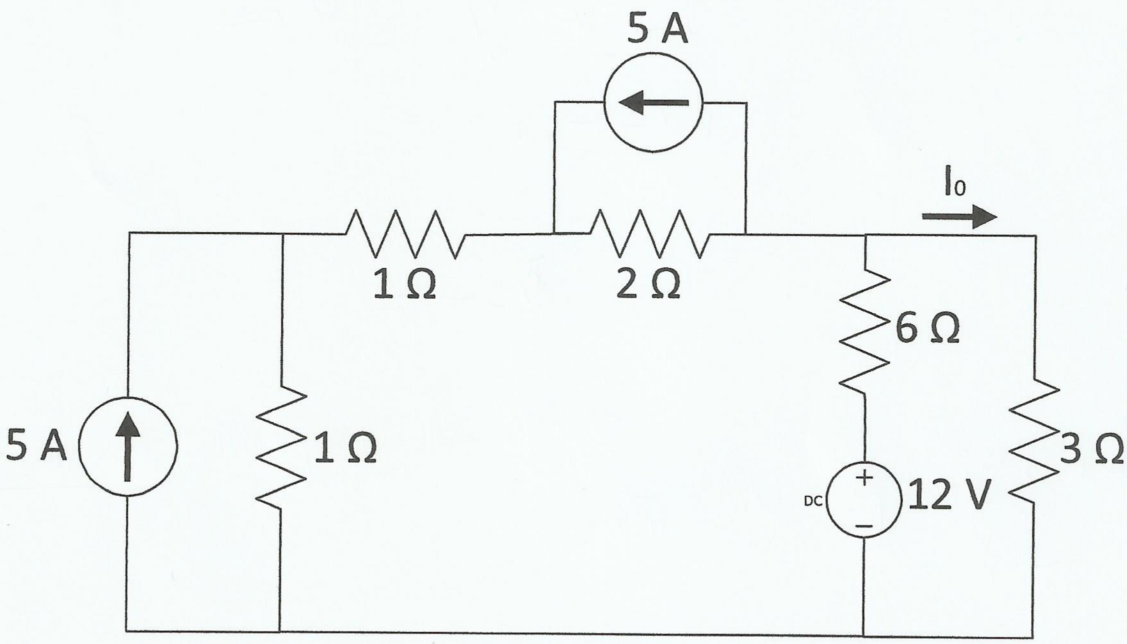
$\Longrightarrow P_{R_L} = 80\% \, P_{max} \Longrightarrow$

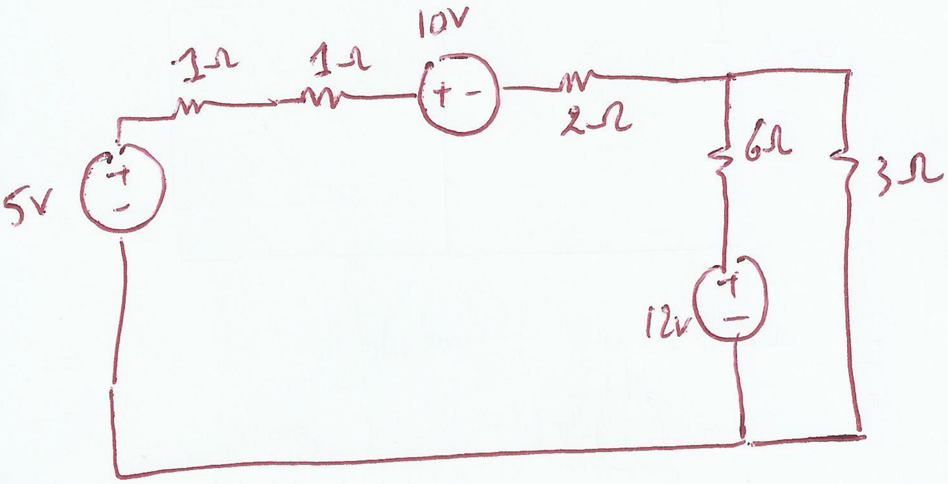
$$\frac{4R_L}{(R_L+4)^2} = 0.8 \times \frac{1}{4} \Rightarrow 20R_L = (R_L+4)^2$$

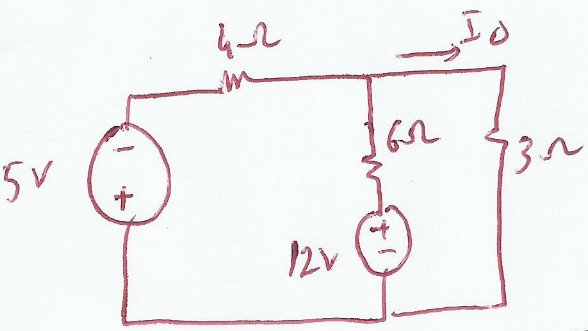
$$\Longrightarrow R_L^2 + 8R_L + 16 = 20R_L \Longrightarrow \begin{cases} R_L = 6 - \sqrt{20}\,\Omega \\ \text{or} \\ R_L = 6 + \sqrt{20}\,\Omega \end{cases}$$

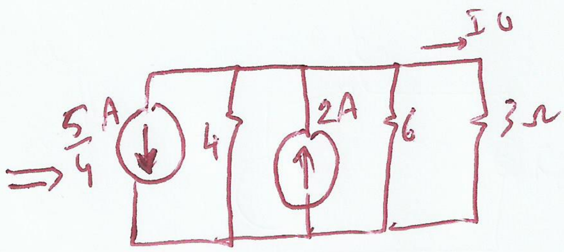
Question 6 [1~~9~~2 points]

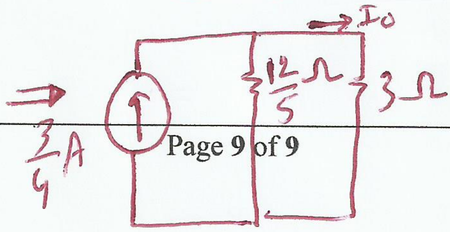
Use source transformation to find I_0 in the network in the below figure.











$$\Rightarrow I_0 = \frac{3}{4} \times \frac{\frac{12}{5}}{\frac{12}{5}+3}$$

$$= \frac{3}{4} \times \frac{12}{27} = \frac{1}{3}\,A$$